New Value of $\pi(17 - 8\sqrt{3})$

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Abstract

There are some mathematical questions in the world that need some attention, of which one of them is to determine the exact value of the π. The present paper deals with finding the value of π by simple geometric methods. The paper considers the ratio of the area of the circle (πr^2) and the square of the radius (r^2) in calculating π and proves that the circle can be converted into a rectangle/square.

Literature reveals that there are several values of π, but the values that are found are approximate values. This paper claims the new value of π as $(17 - 8\sqrt{3})$. This paper deals with the new value of π by using two different geometric methods. It is observed that the value of π obtained by both the methods is the same. Efforts are also made to identify error in current value of π.

Keywords: value of π, errors in π, new value, area of circle.

1. Introduction

The history of π parallels the entire History of Mathematics. According to Lindeman's proof of transcendence of π was one of the highlights of the nineteenth century's is not an easy number and is a ratio of the circumference and the diameter of the circle. π is a number that lies between 3 and 4. The basic properties of π were identified in the period of the classical Greek Mathematics. The literature thus reveals that π has a very long history.

The value of π is significantly explored for its utility in today's life. As we are aware, π is a transcendental number and π plays a vital role in modern technology, right from computer signal processing to medical imaging, GPS navigation to aircraft design. It is essential for analyzing wave patterns in physics and communications, calculating the curves and movements in digital graphics, and is even found in biological processes like heartbeats and DNA structure.

The value of π was discovered around 4000 years, many experts tried to find the exact answer by studying and applying different methods, but so far, the perfect solution has not been achieved. Current value of π is 3.141592653… with the help of a calculator is around 300 trillion digits, yet the value of pi is approximate and cannot be exact. Till date found the oldest records of π, it seems to be a bit controversial to whether it had influenced the Archimedes research. π is defined in two ways:

1. The circumference of a circle divided by its diameter.
2. The area of a circle divided by the square of its radius.

However, in the method of circumference divided by diameter, it is difficult to measure the end points precisely, and it is impossible to obtain the exact value through algebraic methods.This paper includes study of π using the method of area of the circle divided by the square of the radius and also reveals that the area of a circle can be expressed algebraically. If the meaning of π is restricted only to circumference divided by diameter, and if circumference/diameter (c/d) = area/r^2= πr^2 /r^2 is not accepted, then while c/d = 3.14159... may be true, but as per this paper the value of a/r^2= 17 – 8√3 is indeed exact.

Many experts in the past have attempted to find the exact value of π, using methods such as n-sided polygon approximations and infinite series. However, till today, the exact value of π has not been found. There are some common beliefs in the world about π, such as:

The value of π cannot be exact and it is not an algebraic number.

A square with the same area as a circle cannot be constructed, etc. Furthermore, a thorough study includes comparison between the currently accepted value of π and other values discovered by experts and concluded that the value of π, to be $17 - 8\sqrt{3}$. The square, whose area is equal to that of the given circle, is constructed with different efficient algorithms. This construction is based on the values of π as $(17 - 8\sqrt{3})$ which are found in various papers. To support the possibility of construction different methods are used.

The square whose area is equal to that of the area of the given circle is constructed with different efficient algorithms. This construction is based on the values of π $(17 - 8\sqrt{3})$ which are found in various papers. To support the possibility of construction different methods are used. The graphical representation is used here to get accurate results. After a lot of trials with different methods the problem was successfully resolved by using the following steps.

To construct the square of area, which is equal to area of the given circle, we need accurate value of π. But as we know the value of π as 3.14159…which is up to 300 trillion digits. This value is obtained by dividing the polygon into infinite parts and gives us the approximate value therefore construction of square which is equal to area of given circle is impossible with this approximate value of π. The value of π used in this paper is $(17 - 8\sqrt{3})$ which is proved by using the geometric figures and algebraic equations [1-8].

The challenge of squaring the circle is easily solved by the examples below. There is a simple solution to convert the circle into a rectangle. If one calculates the area of the circle and the area of the rectangle.

In Mathematics the values are of two different types:

Exact values & approximate values.

Exact value	**Approximate value**
1/3	**= 0.33333…**
2/3	**= 0.66666…**
√2	**= 1.41421…**
√3	**1.73205…**

One can draw the line of the exact value but rather it is difficult to draw the lines of approximate values. For example, it is easy to determine the 1/3 part of the line or to draw a line of √2 but it is difficult to draw a line with value π. Literature survey reveals that the value of π cannot be exact is due to the method (n sided polygon method) used to find it. This method deals with the polygons with infinite sides, it is assumed that there is no end to this process, and thus the π cannot have an exact value.

To draw a line of length π, it is the distance covered in one revolution of the wheel of the cart (diameter × π) is a line of length of πd. In 'n' rounds (n × diameter × π) is a line of length of (n × diameter × π). Thus, it is possible to draw a line of length π and multiples of π. The table below gives the comparison between the current value of π and the exact value of π.

Comparison between Current value of π & exact value of π

s. r.	Current value of pi π	Exact value of pi π
1	3.1415926… 300(10^{13}) digits number	$(17 – 8\sqrt{3})$ 3.14359353944…
2	Transcendental number	Algebraic number
3	Very hard work but not challenge	Very simple work but challenge
4	Squaring the circle impossible	Squaring the circle possible
5	Approximate results for example $(4 – \pi) + (\pi – 3) = 1$ = 0.8584073… + 0.1415926… = 0.9999999…	Exact results for example $(4 – \pi) + (\pi – 3) = 1$ $= (8\sqrt{3} – 13) + (14 – 8\sqrt{3})$ = 1

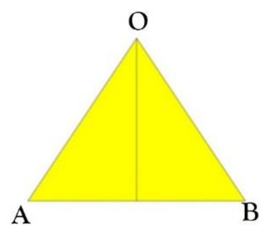
2. Experimental

2.1 Exact Value Of π a Geometrical Approach

This paper deals with finding the values of π by two different methods.

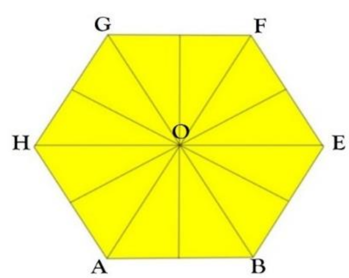
Triangle is the three-sided polygon enclosing the area in their three sides. Thus, let us consider the equilateral triangle which has equal sides and angles.

Fig. 1: Figure shows the triangle under consideration



Height of an equilateral triangle = $\sqrt{3}/2 \times$ side. Area of an equilateral triangle = $(\sqrt{3}/4) \times \text{side}^2$

Figure. 2: Obtained by combining the triangles indicated in figure 1



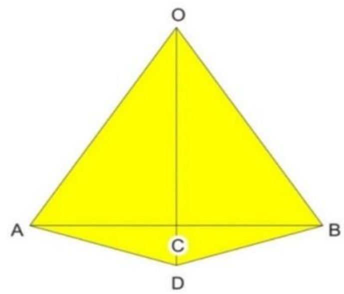
If we combine six equilateral triangles with each other then we get hexagon.

Area of hexagon = 6 × area of an equilateral triangle = $6(\sqrt{3}/4) \times \text{side}^2$.

If we extend the height of the triangle to the length of the side [i.e. height = side of triangle].

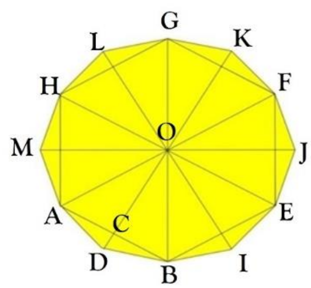
Now join the triangles as shown in figure 3.

Figure 3 If the height of the triangle is extended, we get a kite as shown in the figure.



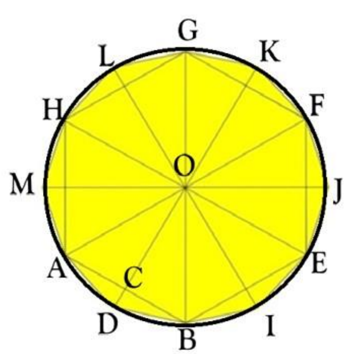
We get kite quadrilateral OABD. Area of the kite = $side^2/2$. If we join six kites, then we get 12 sides polygon.

Figure 4: Obtained by combining 6 kites shown in figure 3



Area of polygon = $6(side^2/2) = 3(side^2)$. Now draw the circle with radius equal to length of side of an equilateral triangle to inscribe the polygon.

Figure 5: Indicates that this polygon formed in figure 4 is inscribed in the circle.



To determine the area of triangle, it's known that area of the polygon drawn within this circle is $3(side^2)$. Therefore, to find the area of circle we need to find the area of remaining area only. There are eight different methods to calculate the area, for which infinite algebraic equations must be considered in this paper.

Area of circle = Inscribed 12 side polygon area + Remaining part of the circle.
= $3r^2$ + Remaining part of the circle.

Figure 6a: indicates the inscribed dodecagon and figure 6b: dividing the figure 6a into 4 parts and then inverting we get figure 6 b

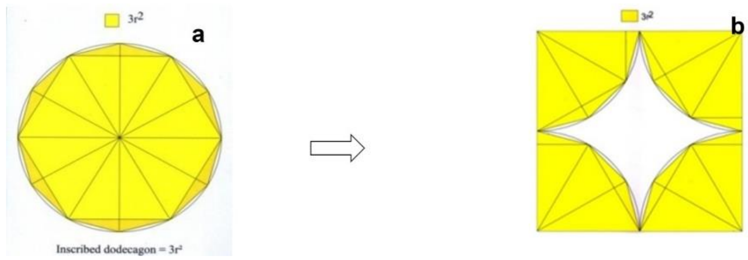
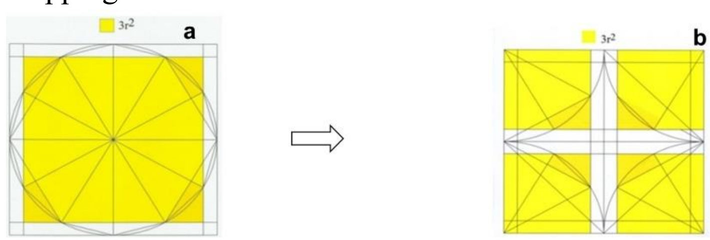


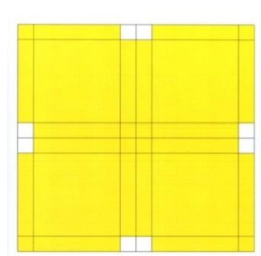
Figure 6a indicates the inscribed dodecagon having the area of $3r^2$. On dividing the above circle into 4 equal parts and joining them as shown them in figure 6b, then it seen that a square is formed with the length of side $(r + r) = 2r$. The area of the square will then be $(2r \times 2r) = 4r^2$. It is found out that the area of the yellow-colored part is $3r^2$. Thus, the area of the white colored part as indicated in the figure 6b is $(4r^2 – 3r^2) = r^2$. The area of white colored part contains a minor part of the circle and some part which belongs to outside the circle. Find the exact area of the remaining part of the circle; by joining some appropriate points to form the below figures.
Figure. 7a: This figure indicates the area of the yellow-colored part is $3r^2$ and 7b is obtained by dividing the circle into 4 parts and then flipping them.



In figure 7a the side of yellow color square is $(\frac{\sqrt{3}r}{2}) + (\frac{\sqrt{3}r}{2}) = \sqrt{3}r$.

Area of yellow color square part $= \sqrt{3}\,r \times \sqrt{3}\,r = 3r^2$.

The area of the remaining white color part of figure 7 is included in area of white colored region indicated in figure 7b. To find this remaining area, divide figure 7 into four equal parts. After overlapπng figures 7a & 7b on each other, we get figure 8.
Figure 8: obtained by overlapping figure 7a and 7b.



As seen in figure 8 white colors eight small squares remain on overlapping figure 7a and 7b. These small squares in figure 8, are the remaining white color part of the circle in figure 7, now Length of side of white square is $\left(1 - \frac{\sqrt{3}}{2}\right) r$. Thus, the area of white square = $\left[\left(1 - \frac{\sqrt{3}}{2}\right) r\right]^2 = \left[1 + \frac{3}{4} - 2\left(\frac{\sqrt{3}}{2}\right)\right] r^2$
$= \left[1.75 - \sqrt{3}\right] r^2$

∴ Area of eight white squares is $= 8\left(1.75 - \sqrt{3}\right) r^2 = (14 - 8\sqrt{3}) r^2$ ∴ Area of circle = $3r^2 + (14 - 8\sqrt{3})\, r^2$
$= (17 - 8\sqrt{3})\, r^2$

2.2 **Method 2**

In this method the area of the circle was determined using the square. The detailed description of the methodology is as given below.

Figure 9: Shows a square inscribed in a circle,

Figure 10: Dividing the figure 8 into four parts and flipping them

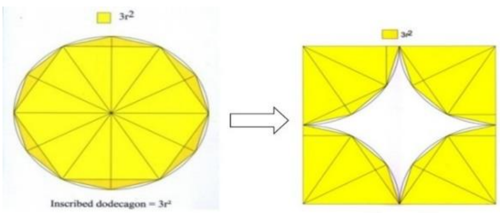


Figure 9 **Figure 10**

In figures 9 and 10, the area of the yellow part is $3r^2$and the white part is a very small portion of the circle. The white part in figure 9 and figure 10 are colored red, and then the figures will be as shown in figure 11 and 12 respectively.

Figure 11: Coloring the white part in figure 9 in red and
Figure 12: Coloring the white part in figure 9 in red

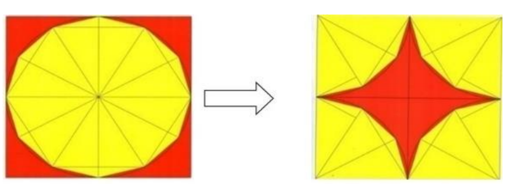


Figure 11 **Figure 12**

In both the above figures the area of the yellow area is $3r^2$ and the remaining red area is $1r^2$. By connecting the appropriate points in figures 11 and 12, figures 13 and 14 are created.

Figure. 13: yellow part has area $3r^2$ and red has area $1r^2$
Figure 14: obtained by dividing figure 13 into 4 equal parts and flipping

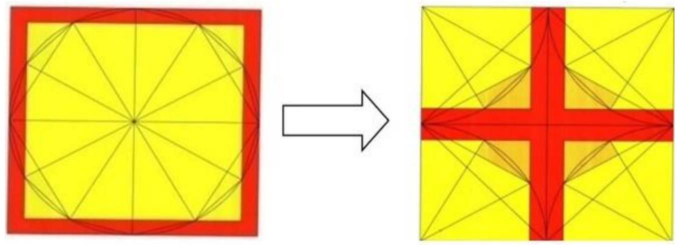


Figure 13 Figure 14

Cut the yellow square having area $3r^2$in figures 13 and 14 and paste the yellow square as shown in the following figures. So, the area of the rest of the circle is in $1r^2$.

Figure 15: obtained by cutting the $3r^2$ yellow part and
Figure 16: obtained by pasting the yellow part cut in figure 13

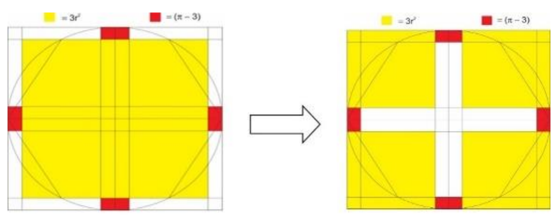


Figure 15 Figure 16

The red part in both the above figures is the part of the circle which was added to the red part. So, its area $(1r - \sqrt{3}/2r)^2 \times 8 = (2r - \sqrt{3}r)^2 \times 2 = (14 - 8\sqrt{3})\ r^2$. Consider another method to prove that the same segment is the reminder of the circle.

Figure 17: Comparing figures 19 and 21 the area of the red colored portion is the same
Figure. 18: Comparing figure 20 and 22 the area of the red colored portion is the same

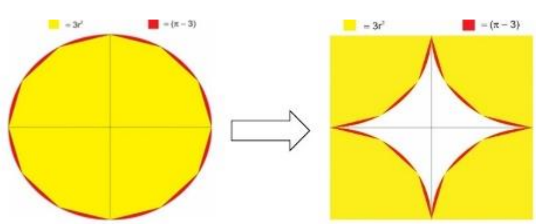


Figure 17 Figure 18

The red part of the circle above is added to the yellow part of the circle below.

Figure 19: Circle inscribed in the square in figure 11 and
Figure 20: figure 19 is divided into 4 parts and flipped

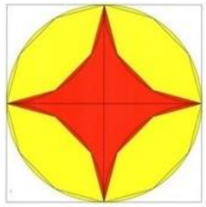


Figure 19

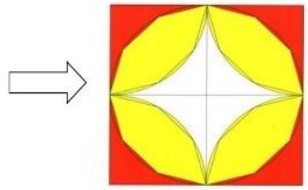


Figure 20

The yellow square in the previous figures were cut out and pasted on the red part to get the area of the yellow part in both the above figures. Here the red part is glued on the opposite yellow part as shown in the figure below. Knowing that the area of the red part is $1r^2$, the area of the remaining yellow part = $[(\sqrt{3} - 1)^2 \times 4] = (16 - 8\sqrt{3})\ r^2 = 2r^2 + (14 - 8\sqrt{3})\ r^2$.
Figure 21: The backside view of figure 19,
Figure 22: The backside view of figure 20,
Figure 23: Indicates red colored part is equal to yellow colored part that is $2r^2$.

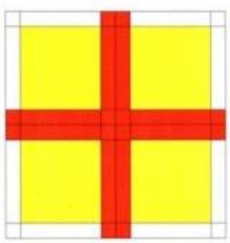


Figure 21

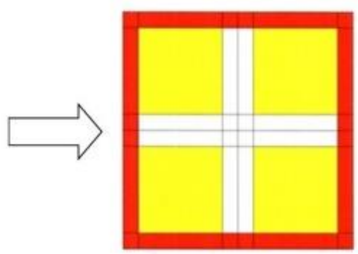


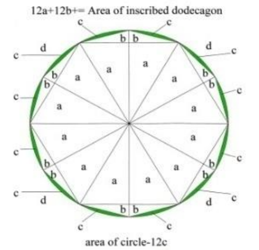
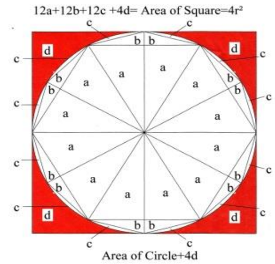
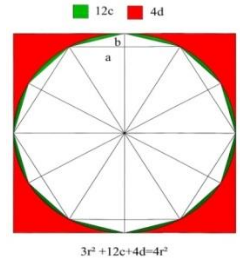
Figure 22

Figure 23

By the first method, the area of the circle = $3r^2 + (14 - 8\sqrt{3})\ r^2 = (17 - 8\sqrt{3})\ r^2$. Area of circle by the second method = $1r^2 + [2 + (14 - 8\sqrt{3})]\ r^2 = (17 - 8\sqrt{3})\ r^2$. This remainder of the circle came to be the same each time.

2.3 ALGEBRAIC PROOFS

Basic information: Note: let a, b, c & d each part shows area in following figures

		
Area of inscribed dodecagon **= (12a + 12b) = 3r²** **= (πr² - 12c)**	**Area of circumscribed square** **= (12a + 12b + 12c + 4d) = 4r²** **= (πr² + 4d)**	**(4 – π) r² = 4d** **(π – 3) r² = 12c** **(4d + 12c) = [(4 – π) + (π – 3)] = 1**

Area of inscribed hexagon = 12a = $(1.5\sqrt{3})\, r^2$ a = $(0.125\sqrt{3})\, r^2$

Area of inscribed dodecagon = (12a + 12b) = $3r^2$ = $(\pi r^2 - 12c)$

(12a + 12b) – 12a = 12b = $(3 - 1.5\sqrt{3})\, r^2$ b = $(0.25 - 0.125\sqrt{3})\, r^2$
Area of circumscribed square = (12a + 12b + 12c + 4d) = $4r^2$ = $(\pi r^2 + 4d)$
Area of circle = (12a + 12b + 12c) = $(3r^2 + 12c)$ = $(4r^2 - 4d)$

$(12c + 4d = 1r^2)$ = $(12c + 4d = 1r^2)/4$ = $(3c + d = 0.25r^2 = a + b)$

The exact area of inscribed dodecagon = $3r^2$. To calculate the exact area of circle, we must calculate the exact area of 12c. Hence there is no need to divide the whole circle into an infinite number of parts to calculate its accurate area. To estimate the exact area of part 12c & part 4d we will consider some equations in which there is no need for 12c or 4d all equations are in teams of a & b. Let us see following examples

Area of inscribed dodecagon = 12(a + b) = $12(0.25)\, r^2$ = $3r^2$
Area of circumscribed square = 16(a + b) = $16(0.25)\, r^2$ = $4r^2$
Area of inscribed square = 8(a + b) = $8(0.25)\, r^2$ = $2r^2$
Area of inscribed triangle = 6a = $6(0.125\sqrt{3})\, r^2$ = $(0.75\sqrt{3})\, r^2$
Area of inscribed hexagon = 12a = $12(0.125\sqrt{3})\, r^2$ = $(1.5\sqrt{3})\, r^2$
Area of circumscribed hexagon = 16a = $16(0.125\sqrt{3})\, r^2$ = $(2\sqrt{3})\, r^2$
Area of circumscribed triangle = 24a = $24(0.125\sqrt{3})\, r^2$ = $(3\sqrt{3})\, r^2$
Area of circumscribed dodecagon = 96b = $96(0.25 - 0.125\sqrt{3})\, r^2$ = $(24 - 12\sqrt{3})\, r^2$
Area of circle = (4a + 68b) = $4(0.125\sqrt{3})\, r^2 + 68(0.25 - 0.125\sqrt{3})\, r^2$
$= (0.5\sqrt{3})\, r^2 + (17 - 8.5\sqrt{3})\, r^2 = (17 - 8\sqrt{3})\, r^2$
$12c = (17 - 8\sqrt{3})\, r^2 - 3r^2 = (14 - 8\sqrt{3})\, r^2$
$3c = (14 - 8\sqrt{3})\, r^2/4 = (3.5 - 2\sqrt{3})\, r^2$
$= (\text{area of circumscribed dodecagon} - 3r^2)/6$
$= [(24 - 12\sqrt{3})\, r^2 - 3r^2]/6$
$= (3.5 - 2\sqrt{3})\, r^2$

$4d = [4r^2 - (17 – 8\sqrt{3})\ r^2] = (8\sqrt{3} – 13)\ r^2$

$d = (8\sqrt{3} – 13)\ r^2/4 = (2\sqrt{3} – 3.25)\ r^2$

$= (\text{area of circumscribed hexagon} – 3r^2) – (d + 3c)$
$= (2\sqrt{3} – 3.25)\ r^2$

Note that x, y is any numbers

Area of (x) inscribed dodecagon + area of circumscribed [2y hexagon + (2x + 3y) dodecagon] – [area of (y) circumscribed square]
$= [(3x + 4y) \times (4a + 68b)] = [(3x + 4y) \times (12a + 12b + 12c)] = [(3x + 4y) \times (17 – 8\sqrt{3})\ r^2]$

<u>Example 1</u>: x = 5, y = 8

Area of 5 inscribed dodecagon + area of circumscribed [{(2 × 8) = 16 hexagons} + {(2 × 5) + (3 × 8) = 34 dodecagon}] – area of 8 circumscribed square =
area of (3 × 5) + (4 × 8) = … (4a + 68b) = … (12a + 12b + 12c) = … $(17 – 8\sqrt{3})\ r^2$
$= 47(4a + 68b) = 47(12a + 12b + 12c) = 47(17 – 8\sqrt{3})\ r^2$

Area of 5 inscribed dodecagon + area of circumscribed [16 hexagon] + {34 dodecagon}] – area of 8 circumscribed square

$= 47(4a + 68b) = (188a + 3196b)$
$= 5(12a + 12b) + 16(16a) + 34(96b) – 8(16a + 16b)$
$= (60a + 60b) + (256a) + (3264b) – (128a + 128b)$
$= 47(4a + 68b)$

$47(12a + 12b + 12c) = (564a + 564b + 564c)$
$= 5(12a + 12b) + 16(12a + 12b + 3c + 2d) + 34(12a + 12b + 18c) – 8(12a + 12b + 12c + 4d)$
$= (60a + 60b) + (192a + 192b + 48c + 32d) + (408a + 408b + 612c) – (96a + 96b + 96c + 32d)$
$= (564a + 564b + 564c)$
$= 47(12a + 12b + 12c)$
$= 5(3r^2) + 16(2\sqrt{3})\ r^2 + 34(24 – 12\sqrt{3})\ r^2 – 8(4r^2)$
$= (15r^2) + (32\sqrt{3})\ r^2 + (816 – 408\sqrt{3})\ r^2 – (32r^2)$
$= (799 – 376\sqrt{3})\ r^2$
$= 47(17 – 8\sqrt{3})\ r^2$

<u>One more Example</u> x = 17 y = 23

Area of 17 inscribed dodecagon + area of circumscribed [(2 × 23) = 46 hexagons + (2 × 17) + (3 × 23) = 103 dodecagons] – area of 23 circumscribed square
= area of (3 × 17) + (4 × 23)
$= 143(4a + 68b) = 143(12a + 12b + 12c) = 143(17 – 8\sqrt{3})\ r^2$

143 circles = Area of 17 inscribed dodecagon + area of circumscribed 46 hexagon + 103 dodecagon – area of 23 circumscribed square

$143(4a + 68b) = (572a + 9724b)$
$= 17(12a + 12b) + 46(16a) + 103(96b) – 23(16a + 16b)$
$= (204a + 204b) + (736a) + (9888b) – (368a + 368b)$
$= 143(4a + 68b)$

$143(12a + 12b + 12c) = (1716a + 1716b + 1716c)$

$= 17(12a + 12b) + 46(12a + 12b + 3c + 2d) + 103(12a + 12b + 18c) – 23(12a + 12b + 12c + 4d)$

$= (204a + 204b) + (552a + 552b + 138c + 92d) + (1236a + 1236b + 1854c) – (276a + 276b + 276c + 92d)$

$= (1716a + 1716b + 1716c)$

$= 143(12a + 12b + 12c)$

$143(17 – 8\sqrt{3})\, r^2$

$= 17(3r^2) + 46(2\sqrt{3})\, r^2 + 103(24 – 12\sqrt{3})\, r^2 – 23(4r^2)$

$= (51r^2) + (92\sqrt{3})\, r^2 + (2472 – 1236\sqrt{3})\, r^2 – (92r^2)$

$= (2431 – 1144\sqrt{3})\, r^2$

$= 143(17 – 8\sqrt{3})\, r^2$

3. Conclusion

The exact value of $\pi = 17 - 8\sqrt{3}$ as proved in the paper above and the approximate value of $\pi = 3.1435935394$ ...are currently used. Thus, there are some errors in the current values of π which have been compared by using the algebraic equations. Thus, on feeding the value as π to the computer generates infinite number of numbers which don't show any sequence.

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